Abstract

The slag volume incorporated to the material balance plays very important role in the blast furnace work evaluating and prognosis. However, the slag volume is not usually measured but this value is calculated from the burden data and from the slag chemical analysis. The inaccuracies of input data used for the computing degrade not only the resulting material balance but also all following calculations. In the article the suggest for the calculation using over-determined systems of linear equations for maximal exploiting the input data and to reduce the influence of input data inaccuracies is described.

The new method proposes the assessing the total flue dust as a sum of the fine dust from iron ores, fluxes and coke, leading to a system of linear equations. Presented method enables to compare the influence of various sorts of coke or other burden components on the total sum of flue dust. Conversely, in the case of unvarying burden composition the observed output data differences could demonstrate changes of the burden mechanical properties. Full exploitation of proposed mathematical separation of the burden dust into three parts could even help to reduce the flue dust appearance. The mathematical method leading to the over-determined system of linear equations was also used there to reduce the influence of input data inaccuracies.

Keywords: blast furnace, material balance, slag quantity

1. INTRODUCTION

All evaluating as well as all prognosis of the blast furnace reduction and heat work are usually based on the total or at least partial material balance calculated for some time interval or related to the unit of the hot metal production or to the unit of the blast furnace burden. The slag volume that is mostly not measured but only calculated is one of the most important parameters of the total material balance evaluating calculation. Commonly used calculations of the slag volume are based on the slag chemical analysis and on incoming volume of some oxides. Unfortunately, the input data inaccuracies can produce significant errors in the calculated resulting slag volume that degrade all following calculations.

In addition to the slag volume calculation the problem of the mathematical separation of the total flue dust into the fine dust from iron ores, fluxes and coke is also described in this article. Obtained results could be used for the short-term evaluating and prognosis of the blast furnace work. The total flue dust volume is mostly measured in long time intervals and this long-term data could be used in sufficiently accurate short-term evaluating and prognosis only in the case of unvarying burden composition.

2. NOTATIONS

All calculations in this article are presented only for the material balance related to 1 kg of the hot metal. Materials (raw materials, products) are denoted by following symbols using small letters to be distinguished from the chemical elements.

- $hm$: hot metal,
- $s$: slag,
- $io$: iron ores,
- $f$: fluxes,
Relationships in this article are written with following symbols:

\[ A[x,y] \] - the mass analysis of the component (compound or element) \( x \) in the material \( y \), e.g. \( A[Fe,c] \) represents analysis of coke in (total) Fe.

\[ B(x,y) \] - the total mass of the component \( x \) in the material \( y \) that belongs to 1 kg of the hot metal, e.g. \( B(Fe,c) \) is the mass of Fe (in kg) inherent in the mass of the coke that belongs to 1 kg of the hot metal. Specially \( B(\Sigma,y) \) is the total mass of the dry material \( y \) that belongs to 1 kg of the hot metal, e.g. \( B(\Sigma,c) \) is the total mass of the coke that belongs to 1 kg of the hot metal. Obviously \( B(\Sigma,hm) = 1 \), \( B(x,hm) = A[x,hm] \), \( B(x,y) = A[x,y]B(\Sigma,y) \).

### 3. THE FINE DUST FROM IRON ORES, FLUXES AND COKE

The total fine dust can be obviously assessed as a sum of the fine dust from iron ores, fluxes and coke. With respect to various parameters of iron ores, fluxes and coke the mass of the dust from 1 kg of iron ores is expected to differ from the mass of the dust from 1 kg of fluxes or from 1 kg of coke, respectively. In what follows these masses are noted as \( v_{io} \), \( v_f \) and \( v_c \).

For the mass of the total fine dust that belongs to 1 kg of the hot metal the following equation holds:

\[
v_{io} \cdot B(\Sigma,io) + v_f \cdot B(\Sigma,f) + v_c \cdot B(\Sigma,c) = B(\Sigma,d)
\]

The system of three equations was suggested in [1] to calculate the parameters \( v_{io} \), \( v_f \) and \( v_c \) from the material balance of some period:

\[
\begin{align*}
v_{io} \cdot B(Fe,io) & + v_f \cdot B(Fe,f) + v_c \cdot B(Fe,c) = B(Fe,d) \\
v_{io} \cdot B(CaO,io) & + v_f \cdot B(CaO,f) + v_c \cdot B(CaO,c) = B(CaO,d) \\
v_{io} \cdot B(C,i) & + v_f \cdot B(C,f) + v_c \cdot B(C,c) = B(C,d)
\end{align*}
\]

This system of equations represents the partial material balance of the mentioned burden and dust components for Fe, CaO and C, e.g. for the elements and compounds that are relatively precisely analyzed (and where Fe is characteristic for the iron ores, CaO for fluxes and C for coke, respectively).

However, the condition number of this system matrix could rapidly increase if the iron ores tend to be alkaline (e.g. for fluxed sinters). In such case the element of the matrix second column tend to zero. Even small inaccuracies of the input data could significantly influence the accuracy of the solution. If \( B(\Sigma,f) = 0 \) than the calculation of \( v_f \) has no sense and remaining unknowns \( v_{io} \) and \( v_c \) can be calculated by system of two equations:

\[
\begin{align*}
v_{io} \cdot B(Fe,io) & + v_c \cdot B(Fe,c) = B(Fe,d) \\
v_{io} \cdot B(C,i) & + v_c \cdot B(C,c) = B(C,d)
\end{align*}
\]

This system matrix is well-conditioned already for common assumptions and there is no problem to solve it. Therefore, the condition number of the matrix of the system (2) indicates which of systems (2) or (3) should by used to calculate \( v_{io} \) and \( v_c \).

However, both of systems of equations (2) and (3) have the same insufficiency – exploiting only small part of data obtained by material analysis. Every even small inaccuracy of input data can significantly influence or even degrade the founded solution.
This mentioned insufficiency could be eliminated when the systems (2) and (3) are extended with additional balance equations for other analyzed components. The system (2) could be modified to following form, for example:

\[
\begin{align*}
v_{io} \cdot B_{Fe,io} + v_f \cdot B_{Fe,f} + v_c \cdot B_{Fe,c} &= B_{Fe,d} \\
v_{io} \cdot B_{CaO,io} + v_f \cdot B_{CaO,f} + v_c \cdot B_{CaO,c} &= B_{CaO,d} \\
v_{io} \cdot B_{C,io} + v_f \cdot B_{C,f} + v_c \cdot B_{C,c} &= B_{C,d} \\
v_{io} \cdot B_{Mn,io} + v_f \cdot B_{Mn,f} + v_c \cdot B_{Mn,c} &= B_{Mn,d} \\
v_{io} \cdot B_{SiO_2,io} + v_f \cdot B_{SiO_2,f} + v_c \cdot B_{SiO_2,c} &= B_{SiO_2,d} \\
v_{io} \cdot B_{MgO,io} + v_f \cdot B_{MgO,f} + v_c \cdot B_{MgO,c} &= B_{MgO,d} \\
\ldots & \ldots \ldots \ldots \\
v_{io} \cdot B(X,io) + v_f \cdot B(X,f) + v_c \cdot B(X,c) &= B(X,d)
\end{align*}
\]

In view of the fact that the number of equations of such extended systems is greater than the number of their unknowns and that their coefficients are determined with certain inaccuracy the modified systems of equations are over-determined and thus generally unsolvable. However, the well-known least-square method enables to calculate generalized solution of over-determined systems of linear equations.

The matrix form of the system (4) can be written as:

\[
\begin{equation}
\begin{pmatrix}
B_{Fe,io} & B_{Fe,f} & B_{Fe,c} \\
B_{CaO,io} & B_{CaO,f} & B_{CaO,c} \\
B_{C,io} & B_{C,f} & B_{C,c} \\
B_{Mn,io} & B_{Mn,f} & B_{Mn,c} \\
B_{SiO_2,io} & B_{SiO_2,f} & B_{SiO_2,c} \\
B_{MgO,io} & B_{MgO,f} & B_{MgO,c} \\
\ldots & \ldots & \ldots \\
B_{X,io} & B_{X,f} & B_{X,c}
\end{pmatrix}
\begin{pmatrix}
v_{io} \\
v_f \\
v_c
\end{pmatrix}
= 
\begin{pmatrix}
B_{Fe,d} \\
B_{CaO,d} \\
B_{C,d} \\
B_{Mn,d} \\
B_{SiO_2,d} \\
B_{MgO,d} \\
\ldots \\
B(X,d)
\end{pmatrix}
\end{equation}
\]

The matrix forms of all considered systems of equations are the same, i.e.

\[
B \cdot v = b \quad ,
\]

where \( B \) is the matrix of the system,

\( v \) is the vector of the unknowns,

\( b \) is the vector of the right-hand sides of equations.

Corresponding generalized solution (if it exists) is equal to the solution of the system of equations

\[
B^T \cdot v = B^T \cdot b \quad ,
\]

i.e.

\[
v = (B^T \cdot B)^{-1} B^T \cdot b \quad .
\]

The condition number of the matrix \( B^T \cdot B \) again enables to decide which system of equations (2) or (3) is better to be extended and used for every concrete calculation.
Every other equation added to the system of equations raises the believability (statistical relevancy) of the system solution. Moreover, using other well-known mathematical methods the information how partial components influence the solution accuracy can be gained and this information can subsequently simplify seeking for the input data errors.

4. THE SLAG VOLUME

The calculation of the specific slag volume $m_s$ can be found in various textbooks. The slag volume calculation as a sum of all its parts is referred e.g. in [2] and [3] where the authors work mainly with such oxides from the blast furnace burden that come to the slag absolutely or in known ratio. Another approach presented in [4] and described by system of equations (9) is based only on some selected oxides and on relevant data of the slag chemical analysis.

\[
\begin{align*}
B \; CaO,ci &= m_v \cdot A \; CaO,s \\
B \; MgO,ci &= m_v \cdot A \; MgO,s \\
B \; SiO_2,ci -2.14 \cdot B \; Si,\text{hm} &= m_v \cdot A \; SiO_2,s \\
B \; Al_2O_3,ci &= m_v \cdot A \; Al_2O_3,s \\
\ldots
\end{align*}
\]

(9)

In the blast furnace the $SiO_2$ fractionally reduce itself from the above mentioned oxides. Thus, the volume of the $SiO_2$ corresponding to the volume of $Si$ in the hot metal must be subtracted from the volume of the total incoming $SiO_2$. The first equation of the system (9) contains the value $A[CaO,u]$ that is worth attention because it is not only a slag analysis on $CaO$ but it represents total $Ca^{2+}$ volume in the slag (i.e. including $Ca^{2+}$ from $CaS$).

Each of equations from the system (9) should ideally produce the same value $m_v$ but solutions of individual equations are usually significantly different. Resulting value $m_v$ is calculated as the average value of individual equations solutions [4]. Instead of simple average used in [4] the weighted average would be worthier to calculate the resulting value $m_v$ but question of individual weights determining would have to be considered. Individual weights should mainly take into account the relative volume of corresponding oxide in the slag (empirical chemical analysis) as well as in the blast furnace burden (the value calculated from empirical chemical analysis). However, each of necessary empirical analysis is realized with certain inaccuracies and in addition could be significantly influenced by the sample selecting, or, some analysis may be not available.

In view of the fact that the analysis of basic slag-making oxides ($CaO$, $MgO$, $SiO_2$, $Al_2O_3$) are mostly sufficiently attended and that the volume of this oxides is considerably greater than volume of other slag-making oxides the sum of these in the final blast furnace burden and in the slag can be advantageously used in the relative (specific) slag volume calculation:

\[
\begin{align*}
B \; CaO,ci &= m_v \cdot A \; CaO,s \\
B \; MgO,ci &= m_v \cdot A \; MgO,s \\
B \; SiO_2,ci -2.14 \cdot B \; Si,\text{hm} &= m_v \cdot A \; SiO_2,s \\
B \; Al_2O_3,ci &= m_v \cdot A \; Al_2O_3,s \\
B \; CaO,ci + B \; MgO,ci + \ldots &= m_v \cdot A \; CaO,s + A \; MgO,s + \ldots
\end{align*}
\]

(10)

In the summary equation the weight of each of input values is equal to its magnitude, i.e. the weight is equal to the volume of the corresponding oxide in the specific burden volume or in the unit slag volume,
respectively. If the total balance of one of mentioned oxides or the balance of the corresponding element was not sufficiently precise then the oxide should be eliminated from the summary relationship.

Instead of the solution of the simple summary equation the generalized solution of the over-determined system of linear equations (9) for one unknown again seems to be worthier. This system has matrix form (the matrix of the system \(B\) is reduced to the column vector \(b\))

\[
b = m_u \cdot a\quad \text{where } b_i = B(X_i, c_i), b_j = A[X_j, s].
\]

The generalized solution of (11) exists and can be expressed as

\[
m_u = (a^T \cdot a)^{-1} \cdot a^T \cdot b
\]

or, respectively,

\[
m_u = \frac{\sum a_i \cdot b_i}{\sum a_i^2}
\]

The last equation represents the calculation of the coefficient \(m_u\) of the linear regression function

\[
b = m_u \cdot a
\]

by the least-square method for known points \([a, b]\).

Presented calculations represent in essence well-known mathematical method that is very often used mainly in the statistics to enable not only full exploitation of available input data but even consideration of these data believability. In contrast to the methods referred in [2] and [3] presented method makes possible to calculate the slag volume even if any input data are not available or these are not sufficiently believable.

5. CONCLUSION

Presented solutions of both mentioned problems (the assessing the total flue dust as a sum of the fine dust from iron ores, fluxes and coke, and, the slag volume calculation) using over-determined systems of linear equations enable full exploitation of all available input data and mathematically represent the problem to find the coefficients of linear regression functions by the least-square method. Every other viewed component added to the system than raises the believability (statistical relevancy) of the resulting system solution and improve the information how partial components influence the solution accuracy whereas such information can subsequently simplify seeking for the input data errors.

LITERATURE


