Abstract:
Investigations of metal materials properties were always based on performing tests, for confirmation of mechanical properties mainly tensile tests were used. However, due to complicated state of stress after neck occurrence, these cannot be used for practical application at industrial forming processes. That’s why plastometric tests (torsion, press) are further utilized, their outcomes are curves of stress-strain dependence.

After applications of large strains tests always result in curves with maximum stress value corresponding to the peak strain value $\varepsilon_p$. Reaching of this strain value induces (dynamic) re-crystallization or start of recovery. It is mostly one single point on the stress-strain curve, which can be defined unambiguously and mathematically described depending on the basic parameters. Single curves may vary in accordance with various ways of plotting, basic parameters (temperature $T$, strain $\varepsilon$ and strain rate $\dot{\varepsilon}$), additional conditions (chemical composition influence and history of the material – residual strain after incomplete re-crystallization or recovery), structure influence (phases portions, grain size), etc. The paper is focused on various methods of peak stress description on the basis of Gleeble plastometer tests results and Plane Strain Compression tests.

Keywords: Stress-strain curve, peak stress, Gleeble plastometric test

1. INTRODUCTION

Efficient forming of material requires understanding of processes, which run in material during hot forming. Emphasis is put on the development of mathematical models, which would be able to predict the curves of stress dependence on deformation under various conditions of forming [1]. The most important value, which must be determined is the so called peak (maximal) strain, at which dynamic re-crystallization starts in the material and its softening takes place till achievement of the steady state, at which equilibrium occurs between hardening and restoration processes (creation and annihilation of dislocations). Authors of the work [2] proposed one of the methods for determining this mathematical value. Theoretical analysis is also completed by a parametric expression of the strain peak value, which is described by the equation

$$W = A \cdot Z^{2p},$$

where $Z$ is the Zener Hollomon's parameter of the temperature compensated strain rate in the form of $Z = \dot{\varepsilon} \cdot \exp(-Q/RT)$, and $W$ is the temperature compensated time $W = t_p \cdot \exp(Q/RT)$.

2. PEAK STRESS (AND PEAK STRAIN)

If we omit the basic Hollomon's power-law function as the truly basic equation for description of only strengthening branch of the examined stress-strain curve in the form
\[ \sigma = k \varepsilon^R \]  \hfill (1)

then the following equations already describe to some extent also the part of the curve after reaching the peak stress value, marked as \( \sigma_p \). This value is at the peak strain \( \varepsilon_p \) usually regarded as the beginning of dynamic re-crystallization. The real critical value \( \varepsilon_c \) is usually reported by many authors as smaller one and we can determine it approximately as \( \varepsilon_c = (0.65 - 0.3) \varepsilon_p \). We present the following equations only to show some of the many variants of more complex description of the dependence \( \sigma = f(T; \varepsilon; \dot{\varepsilon}) \). In each of the following equations the \( \varepsilon_i \) represents the value of general magnitude of deformation, it means also the possibility \( \varepsilon_p \). The persisting problem consists in the fact that it is rather difficult to determine the value mathematically, so this is solved by reading the results from the concrete course of plastometric tests [3, 4].

The basic types are equations with several constants (5 to 9 constants \( Q \)) of the Hensel – Spittel type [5]

\[ \sigma = Q_1 \cdot \exp(Q_2 \cdot T) \cdot \dot{\varepsilon}_i^{Q_3} \cdot \varepsilon_i^{Q_4} \cdot \exp(Q_5 \cdot \varepsilon_i) \]  \hfill (2)

We present CRM equations as an example of practical equation for concrete material [6]

\[ \sigma = k_{SS} \cdot [1 - \exp(-r \cdot \varepsilon)]^{0.5} \]  \hfill (3)

\[ k_{SS} = 8.39 \cdot [1 + (0.032 \cdot Q_1)] \cdot Q_3^{-0.028} \cdot \varepsilon_i^{0.1} \cdot \exp \left( \frac{3537}{T + 273} \right) \]  \hfill (4)

\[ r = 87.59 \cdot (1 + 3.88 \cdot Q_2) \cdot Q_3^{-0.2} \cdot \exp \left( \frac{2240}{T + 273} \right) \]  \hfill (5)

\( Q_1 \) is contents of manganese in [%], \( Q_2 \) is contents of carbon in [%], \( Q_3 \) grain size in [\( \mu m \)], and \( \varepsilon \) is relative strain \( \varepsilon = \frac{h_1 - h_2}{h_1} \), \( h_1 \) and \( h_2 \) represent initial and final height (thickness) of the rolled product, attention – the constants \( Q_1 \) to \( Q_3 \) are valid here only for the equations (3-5)!

The following equations exist in various variants, e.g. equation of Gittins et al. [7-9]

\[ \sigma = Q_1 + Q_2 \cdot \left( Q_3 + Q_4 \cdot \ln \dot{\varepsilon} + Q_5 \cdot \frac{T + 273}{1000} \right) \]  \hfill (6)

Equation of force description:

\[ \sigma = Q_1 \cdot \varepsilon_i \cdot Q_2 \cdot \dot{\varepsilon}^{Q_3} \cdot \exp \left[ \frac{Q_4}{R \cdot (T + 273)} \right] \]  \hfill (7)

Equation for description of hyperbolic sin:

\[ \sigma = Q_1 \cdot \sinh(Q_2 \cdot Z) \cdot Q_5 \cdot \varepsilon_i^{Q_4} \]  \hfill (8)
where: $Z = \dot{\varepsilon}_i \cdot \exp\left(\frac{Q_3}{RT}\right)$ is the Zener – Hollomon’s parameter, $Q_3$ is activation energy in [kJ.mol$^{-1}$], and $R$ is gas constant.

In all the above equations (except for the equations 3-5) $\sigma$ is natural resistance to deformation in MPa and the coefficients $Q_1, Q_2, Q_3, Q_4, Q_5$ are material constants describing strengthening (unless it is specified otherwise at particular equation). It therefore follows from the above equations that exact determination of the peak stress value $\sigma_p$ is difficult.

If we model the stress curve till the peak value – it is possible to describe its course by the strengthening rate $\theta$. In this case it is possible to describe this value as follows [2]:

$$\frac{\delta \sigma}{\delta \varepsilon} = A \varepsilon + C$$  \hspace{1cm} (9)

If this assumption $\varepsilon = \varepsilon_p, \theta = 0, \sigma = \sigma_p$ is valid, then it is possible to write this equation in the form:

$$\frac{\sigma}{\sigma_p} = \left[\frac{\varepsilon}{\varepsilon_p} \left(1 - \frac{\varepsilon}{\varepsilon_p}\right)\right]^c$$  \hspace{1cm} (10)

where $c$ ... parameter depending on the type of material in the form $c = a + bT$

Several authors [10,11] have already tried to describe mathematically critical strain value for initiation of dynamic re-crystallization with use of the curve of dependence of $\theta-\sigma$. It is, however, necessary to take into account apart from the energy accumulated in the material also the kinetics of the running processes. When plotting the curve of dependence $\frac{d\theta}{d\sigma} - \sigma$ is the minimum of this curve, the point at which dynamic re-crystallization starts. This point can be calculated mathematically as the first derivation of $\frac{d\theta}{d\sigma}$ according to $\sigma$. This value is equal to the inflection point on the curve of dependence of $\theta-\sigma$. By modifying the equations it is possible to obtain the final equation for calculation of the critical strain $\varepsilon_c$.

$$R_{\varepsilon} = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_p} = 1 - \frac{1}{2} \sqrt{1 - 2 \frac{e^{(-5 - 12 + 2\sqrt{5^2 - 65})}}{25 - 1}}$$  \hspace{1cm} (11)

The critical strain value is slightly different from the value of the peak strain of the material, it ranges mostly from (0.65 to 0.8) $\varepsilon_p$ [12]. It depends, inter alia, also on the initial grain size (smaller grains mean smaller value of critical deformation due to the higher volume fraction of grain boundaries), and on the alloying elements.

The so called Arrhenius equation is often used for description of the relationship between the strain rate, stress and temperature (especially at high temperatures).

$$\dot{\varepsilon} = B \sigma_p^\gamma \exp\left(-\frac{Q_{\text{def}}}{RT}\right)$$  \hspace{1cm} (12)
where: \( B, n \) … constants
\( Q_{\text{def}} \) … activation energy for hot deformation

Determination of the value \( Q_{\text{def}} \) using the dependency graph \( \ln (\sigma_p) - \frac{1000}{nRT} \) is described in the works [13,14]. The constant \( n \) can be determined by the linear dependence graph \( \ln (\dot{\varepsilon}) \) to \( \ln (\sigma) \).

One of other possibilities for description of actions running in the material and of related material parameters, is description of re-crystallization kinetics by equations of the Sellars-Tegart-Garofalo type [8]. It is therefore possible to write the equation for hot forming in the following manner:

\[
\dot{\varepsilon} = A_1 [\sinh (\alpha \sigma)]^n \exp \left( \frac{Q}{RT} \right)
\]  

(13)

where:

- \( A_1 \) ... pre-exponential factor
- \( \alpha, n \) … parameters of stress sensitivity to the strain rate
- \( Q \) activation energy, \((J/mol)\)
- \( R \) gas constant \( R = 8.314 \ J/(mol \cdot 1K^{-1}) \)

This general equation is further specified for \( \sigma = \sigma_p \). The above equation can be again modified by using the Zener-Hollomon's parameter [15].

\[
Z = \dot{\varepsilon} \exp \left( -\frac{Q}{RT} \right) = A_2 [\sinh (\alpha \sigma)]^m
\]  

(14)

By modifying this relation we can get

\[
\sinh [\alpha \cdot \sigma_p] = \left( \frac{Z}{A_2} \right)^{1/n}
\]  

(15)

and then the form for determination of the peak stress

\[
\sigma_p = \frac{1}{\alpha} \arg \sinh \left[ \frac{\exp (Q/RT)}{A} \right]^{1/n}
\]  

(16)

which can be mathematically expresses also as

\[
\sigma_p = \frac{1}{\alpha} \ln \left( \frac{A}{\exp (Q/RT) / A} \right)^{1/n}
\]  

(17)

For completeness we derived also the peak strain, which is based on the equation \( W_p = A \cdot Z^{\sigma_p} \). We can modify it in the form \( \varepsilon_p = W_p \cdot Z \) and use it for the following linear regression analysis.
This modification leads to the possibility of mathematical notations in the form

\[
\ln \epsilon_p = \ln A_p + (1 - c_p) \cdot \ln \dot{\epsilon} + (1 - c_p) \cdot \frac{Q}{RT}
\]  

(18)

From the derived mathematical relations for the peak strain and peaks stress it is then possible to determine also description of general stress-train curves as

\[
\sigma = \sigma_p \left[ \frac{\dot{\epsilon}}{\dot{\epsilon}_p} \exp \left( 1 - \frac{\dot{\epsilon}}{\dot{\epsilon}_p} \right) \right]^{\frac{1}{n}}
\]  

(21)

3. EXPERIMENT

For mathematical description of the stress-strain curve for the steel marked as CZ1 the results from experiments on the plastometer GLEEBLE on flat samples of 10x15x20 mm were used. Three strain rates, namely 0.1 s\(^{-1}\), 1 s\(^{-1}\) and 10 s\(^{-1}\), were used for plastometric tests. Stress and strain fro these strain rates were measured within the temperature interval from 800 °C to 1200 °C, with a step of 50 °C. The graphs below show the courses at two strain rates. On the basis of successive linear regressions and determination of the constants (A, α, n) the activation energy Q calculated for the final equation for the strain rate (already mentioned equations (13) and (16)) and the values were calculated.

\[
\sigma_{p-c} = \frac{1}{\alpha} \arg \sinh \left[ \frac{\dot{\epsilon} \exp \left( \frac{Q}{RT} \right)^{\frac{1}{n}}}{A} \right]
\]  

(22)

For completeness we calculated also the value of peak strain, again in the form of already mentioned equation (20)
\[ \varepsilon_{p-c} = \dot{\varepsilon}_p a \exp \arg \sinh \left( Y - \frac{X}{T} \right) \]  

(23)

where \( a \) is the slope for various temperatures, \( X \) is the slope, \( Y \) is the point of intersection with the axis \( y \). The exponent \( d \), which is a function of temperature, was determined as an average value and it was substituted into the final equation [25, 32], where \( \sigma_{p-c} \) is calculated peak stress, or \( \varepsilon_{p-c} \) as calculate peak strain.

\[ \sigma = \sigma_{p-c} \left[ \frac{e}{\varepsilon_{p-c}} \exp \left( 1 - \frac{e}{\varepsilon_{p-c}} \right) \right]^d \]  

(24)

The graph in Figure 1 shows the dependence of stress on deformation at one of the three investigated strain rates. These values are experimental. All the curves show a rapid rise to the maximum stress, which is characterized by peak strain and peak stress. This is followed by gradual decline to an equilibrium state. As expected, with the decreasing temperature of deformation the values of peak stress and peak strain, and thus of the steady state increase, and they are characterized by grain refining during dynamic recrystallization [25, 32]. For calculations we used the experimental values. These were only the peak values of strain and stress at the following temperatures: (800, 850, 900, 950, 1000, 1050, 1100, 1150 and 1200)°C at the strain rates (0.1, 1, 10) s\(^{-1}\). For mathematical evaluation we used the entire available range of temperatures and corresponding experimental results.

The graph in Fig. 2 shows a comparison of experimental and calculated values with very good agreement, Fig. 2, which is then reflected also in comparison of the stress-strain curves, Fig. 3.

The constants for establishment of the functional dependence between the experimental and calculated peak values of stress and strain were calculated with use of the above equations. Graphs (2 and 3) show the relationship between the experimental and calculated stress values and plot complete strain curves.
Table 1. Comparison of experimental and calculated results of peak stresses and peak strains for three strain rates and $T = 800 – 1200$ °C

<table>
<thead>
<tr>
<th>$T(˚C)$</th>
<th>0.1s$^{-1}$</th>
<th>1s$^{-1}$</th>
<th>10s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_p$</td>
<td>$\sigma_p$</td>
<td>$\sigma_{pc}$</td>
</tr>
<tr>
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<td>0.20</td>
<td>171</td>
<td>190</td>
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<tr>
<td>850</td>
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<tr>
<td>1200</td>
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</tbody>
</table>

4. CONCLUSIONS

Calculations of peak stress on the strain curves belong to the fundamental mathematical analyses of the forming process at determination of the h resistances to deformation. The used equations, which take into consideration the basic process parameters, such as temperature $T$, strain $\varepsilon$ and strain rate $\dot{\varepsilon}$ (in some cases completed with the effect of the elements, deformation history, cast or formed state, structure, ...) are used for description of the complete curve both in the strengthening and restoration areas. It is difficult to determine the highest of them, i.e. the peak stress as it is necessary to substitute into the equation the experimentally determined peak strain. On the basis of the now classically used Sellars-Tegart-Garofalo equation and known procedures for determination of the activation energy $Q$ it is afterwards possible to determine mathematically the searched value of the peak stress. When we complete this analysis also with the mathematical description of the peak strain and if we merge those values into the final equation, we will obtain also the possibility of determination of complete stress-strain curve. For illustration we present just an example of an experiment with its evaluation without specific values of the material composition and material constants (we present the activation energy at hot deformation of the steel CZ1 at the magnitude $Q = 303,800$ J/mol. calculated by us). The stress-strain curves were plotted for the strain rates (0.1, 1, 10) s$^{-1}$ on the basis of the experimental values, and graphical dependences were constructed for comparison of experimental and calculated peak stress and strain values. The calculated values of the peak strain and peak stress were recorded in Table 1. It is evident from the graphs (2 and 3) that the calculated values are in all cases consistent with the experimental values.

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LITERATURE


